**Exercise Set 5 Portfolio Optimization, Risk and Return Attribution**

In Exercise Set 1, you chose six asset classes for further study. You are to construct several portfolios investing in these six asset classes.

**Question 1:**

Graph the cumulative returns to a $1 investment in all six assets in MATLAB. State both the cumulative dollar returns and their annualized mean returns in a table below the graph.

**Answer:**

Chart

Description automatically generated

**Graph 1**

The cumulative return of 1$ and annualized return of each index is listed in the table below:

|  |  |  |
| --- | --- | --- |
| Return | |  |
| **Asset Name** | **Cumulative Dollar Returns** | **Annualized Mean Returns** |
| SPX | 15.808 | 0.096 |
| USGG10YR | 0.580 | -0.018 |
| MXCN | 1.298 | 0.009 |
| LT09TRUU | 4.188 | 0.049 |
| SPGCITR | 1.461 | 0.013 |
| LUMSTRUU | 3.567 | 0.043 |

**Table 1**

**Question 2:**

2. Our Base case corresponds to the covariance matrix over the entire sample from August 1988 – December 2022. Our Stress Case uses the covariance matrix estimated on the trailing 36-month window ending 31-Jul-2010. Our targeted (monthly) return is 0.0055 (6.8% annually). With this information, solve for six portfolios: three under the Base case and three under the Stress case. The three portfolios are

(i) minimum variance,

(ii) the targeted rate of return portfolio, and

(iii) the risk parity portfolio.

Make tables showing the following and with each table write a brief commentary that contrasts the relevant findings.

a. Table summarizing the portfolios’ weights.

b. Table summarizing the portfolios’ returns. Exclude the minimum variance portfolio from here onward

c. Table summarizing the portfolio risks (w’Vw) ½

d. Table summarizing MCR.

e. Table summarizing the Risk Budgets (risk attribution).

**Answer:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | |  |  |  |  |  |
| **Asset Name** | **wmvBase** | **wmvStress** | **wBase** | **wStress** | **wrpBase** | **wrpStress** |
| SPX | -0.009 | 0.015 | 0.408 | 0.408 | 0.053 | 0.034 |
| USGG10YR | 0.101 | 0.081 | 0.000 | 0.000 | 0.101 | 0.085 |
| MXCN | -0.002 | -0.012 | 0.000 | 0.000 | 0.027 | 0.018 |
| LT09TRUU | 0.299 | 0.342 | 0.592 | 0.592 | 0.433 | 0.516 |
| SPGCITR | -0.004 | -0.014 | 0.000 | 0.000 | 0.040 | 0.023 |
| LUMSTRUU | 0.615 | 0.587 | 0.000 | 0.000 | 0.347 | 0.324 |

**2.a.** Table summarizing the portfolios’ weights.

Table 2

**Note: “wBase” and “wStress” are weights for “MaxSharpeBase” and “MaxSharpeStress” cases respectively.**

**Comment:**

In the minimal variance portfolio for both the base and stress cases, we find that we are shorting several of the assets in our portfolio. This makes sense to ensure minimum variance in returns. In the targeted return portfolio, both the base and stress case involve heavily investing in two assets (SPX & LT09TRUU), and not really investing in any other. This is to hit a higher targeted rate of return. However, the risk parity portfolio (for both cases) reduces overall riskiness by still focusing on investing in those two assets, but with a much greater spread. This contrasts with the targeted rate of return portfolio, which exclusively invests in those two assets.

**2.b.** Table summarizing the portfolios’ returns.

|  |  |  |
| --- | --- | --- |
|  | |  |
| **Portofolio Type** | **Base** | **Stress** |
| mv | 0.003 | 0.003 |
| MaxSharpe | 0.006 | 0.006 |
| riskParity | 0.003 | 0.003 |

**Table 3**

**Comment:**

The returns are as expected for each portfolio. The minimum variance portfolio has the lowest return because it sacrifices returns for less risk. On the other hand, the target rate of return portfolio hits the target rate exactly, while the risk parity portfolio is somewhere in the middle, due to its distribution of risk.

**2.c.** Table summarizing the portfolio risks (w’Vw) ½.

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  |  |
| **MaxSharpBase** | **MaxSharpeStress** | **RiskParityBase** | **RiskParityStress** |
| 0.068 | 0.046 | 0.029 | 0.016 |

**Table 4: Portofolio Risks**

**Comment:**

The risks also match up with what we know about the allocations for each portfolio. The risk parity portfolio has lower risk in both cases, as the risk parity method distributes risk. Additionally, risk is lower for the stress case than the base case for both portfolios.

**2.d.** Table summarizing MCR.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | |  |  |  |
| **Asset Name** | **MaxSharpBase** | **MaxSharpeStress** | **RiskParityBase** | **RiskParityStress** |
| SPX | 0.125 | 0.101 | 0.09 | 0.076 |
| USGG10YR | -0.108 | -0.039 | 0.047 | 0.031 |
| MXCN | 0.124 | 0.136 | 0.179 | 0.141 |
| LT09TRUU | 0.028 | 0.008 | 0.011 | 0.005 |
| SPGCITR | 0.042 | 0.111 | 0.121 | 0.114 |
| LUMSTRUU | 0.017 | 0.008 | 0.014 | 0.008 |

**Table 5**

**Comment:**

In both the stress cases, the MCR is more evenly distributed among every asset for the risk parity portfolio. This makes sense in the context of the weights of either portfolio. The targeted rate of return portfolio invested more heavily in a few assets while the risk parity portfolio spread out investment. Does, the MCR was more evenly distributed for the risk parity portfolio but concentrated in a certain few asset for the targeted rate of return portfolio.

**2.e.** Table summarizing Risk Attribution.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | |  |  |  |
| **Asset Name** | **MaxSharpBase** | **MaxSharpeStress** | **RiskParityBase** | **RiskParityStress** |
| SPX | 0.051 | 0.041 | 0.005 | 0.003 |
| USGG10YR | 0 | 0 | 0.005 | 0.003 |
| MXCN | 0 | 0 | 0.005 | 0.003 |
| LT09TRUU | 0.016 | 0.005 | 0.005 | 0.003 |
| SPGCITR | 0 | 0 | 0.005 | 0.003 |
| LUMSTRUU | 0 | 0 | 0.005 | 0.003 |

**Table 6**

**Comment:**

Unsurprisingly, the risk budget is reflective of the broader trends already identified. The risk parity portfolio has an equally distributed risk budget while the targeted rate of return portfolio has a risk budget concentrated on certain assets designed to offer a higher potential return.

**Question 3:**

Estimate VaR at the 1-percent level for each of the four portfolios. Tell me exactly what these mean for the three portfolios under both the Base and Stress cases.

**Answer:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | |  |  |  |
| **U/L Bound** | **MaxSharpBase** | **MaxSharpeStress** | **RiskParityBase** | **RiskParityStress** |
| Upper Bound | -0.107 | -0.050 | -0.035 | 0.000 |
| Lower Bound | 0.243 | 0.187 | 0.115 | 0.082 |

**Table 7**

**Comment:**

Based on the table above, we are 99% confident that the annualized expected return for each portfolio is as follows:

1. For the target return portfolio in base case, we are 99% confident that the annualized return falls between -10.7% and 24.3%.
2. For the target return portfolio in stress case, we are 99% confident that the annualized return falls between -5.04% and 18.7%.
3. For the risk parity return portfolio in base case, we are 99% confident that the annualized return falls between -3.48% and 11.5%.
4. For the risk parity return portfolio in stress case, we are 99% confident that the annualized return falls between 0.0057% and 8.25%.

**Question 4:**

Now, let’s concentrate on the base case targeted return portfolio. First, construct this portfolio’s vector of returns. For example, if M = [tbl.X1,…, tbl.X6] and wBase ~ 6 x 1 vector of weights, then portRet = M\*w. Here portRet are the Base case targeted returns. Now do the following:

a. Using the BlackRock factors and a 36-month trailing window, estimate and plot the betas (factor loadings) as time series.

b. Estimate and plot the monthly return attribution having solved for the betas.

**Answer:**

**4.a.** Using the BlackRock factors and a 36-month trailing window, estimate and plot the betas (factor loadings) as time series.

Graphical user interface, diagram

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**Graph 2**

**4.b.** Estimate and plot the monthly return attribution having solved for the betas.

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**Graph 3**

**Question 5:**

Remark on any outstanding behavior in these beta plots, especially over previous recessions.

**Answer:**

The betas indicate significance of each factor to returns. If we look at the shaded area (recessions), we can see that the Economic factor becomes much more important, as does real rates. On the other hand, the importance of inflation and emerging markets drop drastically, as a reflection of the economic conditions of a recession.

**Question 6:**

Remark on any outstanding returns behavior in the attribution plots, especially over previous recessions. (Ignore alpha and the idiosyncratic return).

**Answer:**

The attribution plots show similar behavior around recessions, in regard to the importance of real rate and the economic factor. For both, we see a fall in attribution at the start of a recession and then a large spike upwards as a recovery begins. This fits with what we know of the business cycle and how a recession occurs. Inflation displays the opposite pattern in its attribution plot, having a greater attribution value initially and then falling towards the end. This is probably reflective of the slowdown from a recession putting a dampener on the economy, which makes inflation increasingly important, with the reverse happening during the recovery from a recession.

--------------------------------------------**The End of the Exercise Set 5**---------------------------------------